

# Generalizing Planck's law: Nonequilibrium emission of excited media

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Using a quantum-kinetic many-body approach, exact results for the interacting system of field and matter in a specified geometry are presented. It is shown that both the spectral function of photons and the field fluctuations split up into vacuum- and medium-induced contributions, for which explicit expressions are derived. Using Poynting's theorem, the incoherent emission is analyzed and related to the coherent absorption as may be measured in a linear transmission-reflection experiment. Their ratio defines the medium-induced population of the modes of the transverse electromagnetic field and so generalizes Planck's law to an arbitrarily absorbing and dispersing medium in a nonequilibrium steady state. For quasi-equilibrium, this population develops into a Bose distribution whose chemical potential marks the crossover from absorption to gain and, also, characterizes the degree of excitation. Macroscopic quantum phenomena such as lasing and quantum condensation are discussed on this footing.

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## INTRODUCTION

Since the establishment of Kirchhoff's law till today, emission and absorption are the fundamental observables in spectroscopic investigations. Assuming both ideal thermal radiation outside and thermodynamic equilibrium inside a medium and, also, between outside and inside, Kirchhoff's law  $e(\omega) = n(\omega, T)a(\omega)$  fixes the ratio of emission  $e$  to absorption  $a$  as a universal function  $n$  of frequency  $\omega$  and temperature  $T$ . Thus, emission can be traced back to (i) absorption being observable in transmission-reflection experiments using classical (coherent) light, and (ii) the spectral distribution  $n$  of thermal light, which later was found to be a Bose function by Planck. However, if the very restricting assumption of complete thermodynamical equilibrium is removed, the emission of a medium is to be considered as a pure quantum effect, and quantum-optical and quantum-kinetic methods have to be used to describe the interacting many-body system of field and matter. Using the Keldysh technique for the photon Green's function (GF) [1] and neglecting effects of spatial dispersion (SD), a formula for the intensity emitted by a steadily excited semiconductor into the surrounding vacuum was derived in ref. [2].

In this article this result will be generalized in the following fundamentally important aspects: (a) Rigorous results for the emission will be given in terms of the exactly defined polarization function of an isotropic but otherwise arbitrary medium as an energetically open and spatially inhomogeneous finite system. Hence, this approach applies independently of any further specific material properties or models and, besides others, also SD is exactly considered. (b) Instead of an (electromagnetic) vacuum, an arbitrary nonequilibrium distribution of photons is assumed to be present. In this way a situation will be addressed where neither the radiation nor

the medium are in equilibrium but in different steadily excited states arbitrarily far from equilibrium. (c) Absorption as measured by a linear transmission-reflection experiment is investigated on the same footing and its relation to emission is shown.

## GENERAL THEORY

We assume steady state conditions and start with the field operator of the vector potential in Coulomb gauge, which obeys the inhomogeneous wave equation

$$\left[ \Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \hat{\mathbf{A}}(\mathbf{r}, t) = -\mu_0 [\hat{\mathbf{j}}(\mathbf{r}, t) + \mathbf{j}_{\text{ext}}(\mathbf{r}, t)] \quad (1)$$

and the equal time commutation rules

$$\left[ \hat{\mathbf{A}}(\mathbf{r}, t), \frac{\partial}{\partial t} \hat{\mathbf{A}}(\mathbf{r}', t) \right] = \frac{i\hbar c}{\varepsilon_0} \overleftrightarrow{\delta}(\mathbf{r} - \mathbf{r}'), \quad (2)$$

where  $\overleftrightarrow{\delta}$  is the transverse delta function tensor. In eq. (1), the total transverse current density operator is split into the current density operator  $\hat{\mathbf{j}}$  of the medium and an externally controlled current density represented by given c-number functions  $\mathbf{j}_{\text{ext}}(\mathbf{r}, t)$ .

The Keldysh components of the photon GF

$$\begin{aligned} D_{ij}^>(\mathbf{r}, \mathbf{r}', t - t') &= D_{ji}^<(\mathbf{r}', \mathbf{r}, t' - t) \\ &= \frac{1}{i\hbar} \left[ \langle \hat{A}_i(\mathbf{r}, t) \hat{A}_j(\mathbf{r}', t') \rangle - \langle \hat{A}_i(\mathbf{r}, t) \rangle \langle \hat{A}_j(\mathbf{r}', t') \rangle \right], \end{aligned} \quad (3)$$

describe the field-field fluctuations. Formally solving the Dyson equation for these components, one obtains the so-called *optical theorem* after Fourier transformation with respect to  $t - t' \rightarrow \omega$

$$\begin{aligned} D_{ij}^{\geq}(\mathbf{r}, \mathbf{r}', \omega) &= \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \times \\ &D_{ik}^{\text{ret}}(\mathbf{r}, \mathbf{r}_1, \omega) P_{kl}^{\geq}(\mathbf{r}_1, \mathbf{r}_2, \omega) D_{lj}^{\text{adv}}(\mathbf{r}_2, \mathbf{r}', \omega), \end{aligned} \quad (4)$$

where  $P_{kl}^{\geq}$  are the Keldysh components of the polarization function, which is given by the functional derivative (in compact notation:  $P = \delta j / \delta A$ ) of the induced (averaged) current density  $\mathbf{j} = \langle \hat{\mathbf{j}} \rangle$  to the mean field  $\mathbf{A} = \langle \hat{\mathbf{A}} \rangle$ .

Throughout this paper,  $P = \delta j / \delta A$  is to be taken at  $A = 0$ , i.e., in the linear approximation. This is evident since linear absorption is addressed, on the one hand, and since the incoherent emission is defined as the one without any incident classical field, i.e.,  $\mathbf{A}(\mathbf{r}, t) = 0$ , on the other hand.

## SLAB GEOMETRY

A slab of thickness  $L$  will be considered, which is infinitely extended and homogeneously excited in the transverse  $y$ - $z$ -direction. For notational simplicity, TE-polarized light propagating freely in the transverse direction is considered. Due to cylindrical symmetry around the  $x$ -axis, the transverse vector potential  $\hat{\mathbf{A}}(\mathbf{r}, t)$  can be chosen in the  $z$ -direction.

## CLASSICAL FIELD PROPAGATION

Assuming steady state conditions, the propagation equation for the average field, after Fourier transforming with respect to  $(y, z) \rightarrow \mathbf{q}_{\perp}$  and  $t - t' \rightarrow \omega$ , in this geometry has the structure

$$\int dx' D^{\text{ret}, -1}(x, x') A(x') = -\mu_0 j_{\text{ext}}(x), \quad (5)$$

$$D^{\text{ret}, -1}(x, x') = \left( \frac{\partial^2}{\partial x^2} + q_0^2 \right) \delta(x - x') - P^{\text{ret}}(x, x').$$

Here and in what follows, the variables  $\mathbf{q}_{\perp}$  and  $\omega$ , which enter all equations parametrically only, are omitted where possible. The wave vector in vacuum is  $q_0^2 = [(\omega + i\delta)/c]^2 - q_{\perp}^2$ , and  $D^{\text{ret}, -1}$  is the inverse of the retarded photon GF. The retarded polarization function of the medium  $P^{\text{ret}}(x, x', q_{\perp}, \omega)$  is related to the linear susceptibility  $\chi$  according to  $P^{\text{ret}}(x, x') = -\omega^2 \chi(x, x')/c^2$ . It reflects the translational invariance in transverse directions and enables to exactly include SD.

For a traditional transmission-reflection experiment, the external source on the right-hand side of (5) is to be put zero, and instead an external wave incoming from left or right is assumed. Consequently, the homogeneous propagation equation (5) has two linearly independent solutions, which give the solution of the reflection-transmission problem for incidence from left or right. These solutions are fixed by their asymptotics and there is no need to impose any further boundary conditions on them (even not Maxwell's), since the polarization of the medium increases continuously in the transition region

from vacuum to medium and, consequently, the solutions of (5) evolve continuously, too, from the asymptotic ones (compare also the discussion in [4]). Only if one assumes an abrupt switch of the polarization at the surface, one has to make sure the continuity of the solutions of (5) by imposing Maxwell's boundary conditions.

## TRANSMISSION AND REFLECTION

In the following it is assumed that up to a negligible error the surface of the medium, i.e., the length  $L$  of the slab, can be fixed in a way that any polarization vanishes outside and, hence, any Keldysh component of the polarization function vanishes outside, too. Then the forward propagating solution of the homogeneous equation (5) has the structure

$$A(x) = \begin{cases} e^{iq_0 x} + r e^{-iq_0 x} & \text{for } x < -\frac{L}{2} \\ t e^{iq_0 x} & \text{for } x > \frac{L}{2}, \end{cases} \quad (6)$$

where  $r, t$  are reflection and transmission coefficients for the field amplitudes. Due to the symmetry  $x \leftrightarrow -x$ ,  $A(-x)$  is a solution, too (backward propagating, i.e., incidence from right). The solutions inside need not to be specified. Neglecting spatial dispersion, i.e., assuming  $P^{\text{ret}}(x, x') = P^{\text{ret}} \cdot \delta(x - x')$ , yields inside  $A(x) = a e^{iqx} + b e^{-iqx}$ , where the wave vector is given by  $q^2 = q_0^2 - P^{\text{ret}}$ . In this case, the results of ref. [2] can be reproduced.

Many attempts have been made to consider SD by tracing back this problem to the one of the bulk limit, where due to full spatial homogeneity the polarization functions  $P(\mathbf{q}, \omega)$  can be handled. Most common is the use of bulk (polariton) solutions for the waves (6) inside. This, however, is not consistent with eq. (5) and introduces a high degree of arbitrariness into the problem, since additional boundary conditions (ABC's) [3] have to be used. The dielectric approximation [4] uses the bulk susceptibility inside. It is free of arbitrariness but thought to conflict with energy conservation [10]. The present author suggested to construct solution (6) inside as induced by sources in a surface region [5]. This procedure is free of arbitrariness if the surface region is very small and the sources can be handled as  $\delta$  functions. This condition is obviously fulfilled in some specific materials, where this approach worked well [6]. However, for excitons in semiconductors it was shown by microscopic investigations [7] that all the mentioned proposals fail to reproduce spectroscopic details correctly.

## POYNTING'S THEOREM FOR CLASSICAL FIELDS

Now Poynting's theorem will be exactly addressed on the grounds of eqs. (5,6) only, i.e., without referring to

any of the above mentioned approaches that consider SD approximatively. Quite generally it reads

$$\frac{\partial}{\partial t} \int U(\mathbf{r}, t) dV + \int \mathbf{S}(\mathbf{r}, t) d\mathbf{f} = - \int W(\mathbf{r}, t) dV. \quad (7)$$

Here, the change of the field energy  $U$  will not contribute to any of the quantities discussed below and, hence, can be omitted. Moreover, Poynting vector  $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$ ,  $\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}$ ,  $\mathbf{B} = \text{curl } \mathbf{A}$ , and the absorption  $W = \mathbf{j} \cdot \mathbf{E}$  have been introduced, where the current density defines via  $\mathbf{j} = \frac{\partial \mathbf{P}}{\partial t}$  the polarization  $\mathbf{P} = \chi \mathbf{E}$ .

Integrating over the slab in eq. (7) and Fourier transforming with respect to  $t \rightarrow \omega$ , this simplifies to

$$S\left(\frac{L}{2}, \omega\right) - S\left(-\frac{L}{2}, \omega\right) = - \int_{-\frac{L}{2}}^{\frac{L}{2}} dx W(x, \omega) \quad (8)$$

for the geometry investigated here. In eq. (8),  $S(x, \omega)$  is the  $x$ -component of the Poynting vector.

### COHERENT ABSORPTION

At first, Poynting's theorem (8) will be addressed for average fields (classical light). Assuming a monochromatic wave of frequency  $\omega_0$  incident in the  $(q_0, \mathbf{q}_{\perp,0})$ -direction, i.e.,  $A(x, \mathbf{q}_{\perp}, \omega) = \frac{1}{2}[A_0(x, \mathbf{q}_{\perp,0}, \omega_0)\delta(\omega - \omega_0)\delta_{\mathbf{q}_{\perp}, \mathbf{q}_{\perp,0}} + A_0^*(x, \mathbf{q}_{\perp,0}, \omega_0)\delta(\omega + \omega_0)\delta_{\mathbf{q}_{\perp}, -\mathbf{q}_{\perp,0}}]$ , the static part  $\propto \delta(\omega)$  of the energy balance (8) can be regarded separately. Using this ansatz and (6) in (8) yields after straightforward calculation for each  $\omega_0 \rightarrow \omega$  and  $\mathbf{q}_{\perp,0} \rightarrow \mathbf{q}_{\perp}$

$$1 - |r|^2 - |t|^2 = a = \frac{i}{2q_0} \int dx dx' A^*(x) \hat{P}(x, x') A(x'). \quad (9)$$

On the RHS, due to well-known GF identities [1]

$$\hat{P} = P^{\text{ret}} - P^{\text{adv}} = P^> - P^<, \quad (10)$$

the coherent absorption ( $a > 0$ ) or gain ( $a < 0$ ) balances generation  $iP^>(x, x')$  and recombination  $iP^<(x, x')$  of excitations in the medium, whereas the LHS balances the incoming intensity ( $\propto 1$ ) into absorption and the sum of the reflected and transmitted intensity. The latter may be regarded as the intensity re-emitted coherently to the incoming light and is to be contrasted with the incoherent (or correlated) emission, which will be addressed now.

### POYNTING'S THEOREM FOR FIELD FLUCTUATIONS

The incoherent emission is defined as the one without external sources, i.e., for vanishing average fields. In this

case, Poynting's theorem is still given by eq. (7), but due to the non-commuting field operators ( $\hat{\mathbf{E}}$  and  $\hat{\mathbf{B}}$  as well as  $\hat{\mathbf{j}} = \hat{\mathbf{j}}_0 - e\hat{\mathbf{A}}$  and  $\hat{\mathbf{E}}$ ) the symmetrized Poynting vector  $\hat{\mathbf{S}} = \frac{1}{2\mu_0}(\hat{\mathbf{E}} \times \hat{\mathbf{B}} - \hat{\mathbf{B}} \times \hat{\mathbf{E}})$  and the energy dissipation  $\hat{W} = \frac{1}{2}(\hat{\mathbf{j}}\hat{\mathbf{E}} + \hat{\mathbf{E}}\hat{\mathbf{j}})$ , respectively, have to be used. Their quantum statistical averages are given by the Keldysh components of the photon GF (3).

For slab geometry and TE polarization, the quantum statistical averages of both the Poynting vector  $\hat{\mathbf{S}}$  and the energy dissipation  $\hat{W}$  are defined via  $(\omega, \mathbf{q}_{\perp})$ -integrals (for details of the derivation see, e.g., ref. [2]). In the following, these integrals will be omitted for notational simplicity, and their spectrally and directionally resolved contributions (both scaled by  $1/\hbar\omega$ ) for slab geometry denoted by  $s$  and  $w$  (in contrast to coherent absorption  $a$ ), respectively, will be considered instead:

$$s(x) = -\frac{1}{\mu_0} \left\{ \frac{\partial}{\partial x'} [D^>(x, x') + D^<(x, x')] \right\}_{x'=x}, \quad (11)$$

$$w(x) = \frac{1}{2} \int dx' [P^>(x, x')D^<(x', x) - P^<(x, x')D^>(x', x)]. \quad (12)$$

$D^{\gtrless}(x, x', \mathbf{q}_{\perp}, \omega)$  and  $P^{\gtrless}(x, x', \mathbf{q}_{\perp}, \omega)$  are the Keldysh components of the photon GF and of the polarization function, respectively. Their parametric dependence on the variables  $\mathbf{q}_{\perp}, \omega$  is, as before, not explicitly written in the equations. Note that both the incoherent (or correlated) energy flow  $s$  and dissipation  $w$  are time-independent in the steady state, in contrast to the classical case. The  $\omega$ - and  $\mathbf{q}_{\perp}$ -integrals omitted here indicate merely that field fluctuations of all frequencies and directions contribute to them.

### MEDIUM- AND VACUUM-INDUCED CONTRIBUTIONS

The polarization function

$$P^{\gtrless}(x, x') = P_m^{\gtrless}(x, x') - i\delta \frac{4\omega}{c^2} n^{\gtrless} \delta(x - x') \quad (13)$$

comprises besides the medium part an infinitely weak ( $\delta \rightarrow 0$ ) part which describes the vacuum in absence of the medium and ensures the correct vacuum limit for the *optical theorem* (4) [2].  $n(\mathbf{q}_{\perp}, \omega) = n^< = n^> - 1$  describes a given nonequilibrium distribution of photons owing to external preparation as, e.g., by an appropriate enclosure (heath bath) or incoherent radiation incident from outside. Therefore  $n$  in the following will be referred to as the distribution of the externally controlled or simply of external photons. As a consequence, the *optical theorem* (4) comprises a medium-induced and a vacuum-induced contribution according to

$$D^{\gtrless}(x, x') = D_m^{\gtrless}(x, x') + n^{\gtrless} \hat{D}_0(x, x'), \quad (14)$$

where

$$D_m^{\geq}(x, x') = \int d\bar{x} d\bar{x}' D^{\text{ret}}(x, \bar{x}) P_m^{\geq}(\bar{x}, \bar{x}') D^{\text{adv}}(\bar{x}', x') \quad (15)$$

and

$$\hat{D}_0(x, x') = -i\delta \frac{4\omega}{c^2} \int d\bar{x} D^{\text{ret}}(x, \bar{x}) D^{\text{adv}}(\bar{x}, x') . \quad (16)$$

Correspondingly, the spectral function  $\hat{D} \equiv D^{\text{ret}} - D^{\text{adv}} = D^> - D^<$  decomposes into a medium-induced and a vacuum-induced contribution, too,

$$\hat{D}(x, x') = \hat{D}_m(x, x') + \hat{D}_0(x, x') , \quad (17)$$

where

$$\hat{D}_m(x, x') = \int d\bar{x} d\bar{x}' D^{\text{ret}}(x, \bar{x}) \hat{P}_m(\bar{x}, \bar{x}') D^{\text{adv}}(\bar{x}', x') . \quad (18)$$

Note that  $\hat{D}_0$  is to be contrasted with the spectral function of the pure vacuum  $\hat{D}_{\text{vac}}(x, x') = \cos[q_0(x - x')]/2iq_0$ . Also, it contains an improper integral diverging as  $1/\delta$ , which is compensated by the prefactor  $\delta \rightarrow 0$ . Therefore, evaluating the  $\bar{x}$ -integral for  $\hat{D}_0$ , it is sufficient to regard  $|\bar{x}| > L/2$ , where  $D^{\text{adv}}(\bar{x}, x') = D^{\text{ret}}(x', \bar{x})^*$  is given in terms of solution (6) as

$$D^{\text{ret}}(x, x') = \frac{\Theta(x - x')A(x)A(-x')}{2iq_0t} + \dots x \leftrightarrow x' \dots \quad (19)$$

Inserting (19) in (16) yields

$$\hat{D}_0(x, x') = \frac{1}{2iq_0} [A(x)A^*(x') + A(-x)A^*(-x')] . \quad (20)$$

Using (13) - (20) in (12) for  $\omega > 0$  yields for the energy dissipation

$$w = \int dx w(x) = \frac{i}{q_0} [n^<\mathfrak{P}^> - n^>\mathfrak{P}^<], \quad (21)$$

where the global generation/recombination  $\mathfrak{P}^{\geq}(\omega, \mathbf{q}_{\perp})$  are defined as

$$\mathfrak{P}^{\geq} = \int dx dx' A^*(x) P_m^{\geq}(x, x') A(x') . \quad (22)$$

It is noteworthy that, inserting (14) in (12), all the contributions induced by the medium according to (15) cancel exactly and, thus, contribute neither to the (incoherent) energy dissipation nor to the emission.

Equation (21) balances globally optical excitation  $i\mathfrak{P}^>$  of the medium accompanied by absorption of external photons ( $\propto n$ ) and recombination  $i\mathfrak{P}^<$  accompanied by emission of photons ( $\propto [1 + n]$ , i.e., spontaneous and externally stimulated emission).

## NONEQUILIBRIUM PHOTON DISTRIBUTION

As usual [1], a distribution  $b(\omega, \mathbf{q}_{\perp}) \equiv b^< = b^> - 1$  will be attributed to the global generation/recombination  $\mathfrak{P}^{\geq}(\omega, \mathbf{q}_{\perp})$  in (22) by definition

$$\mathfrak{P}^{\geq} = b^{\geq} \hat{\mathfrak{P}} , \quad (23)$$

where  $i\hat{\mathfrak{P}} = i(\mathfrak{P}^> - \mathfrak{P}^<) = 2q_0a$  is directly related to the coherent absorption  $a$  in eq. (9). In contrast to  $n$ , the occupation  $b$  characterizes globally the distribution of the medium-induced (transverse) optical excitations (e.g., polaritons in semiconductors) over the absorption/gain spectrum.

Using (23) in (14), e.g., for  $(x, x') > L/2$ , the field fluctuations take the explicit form

$$D^{\geq}(x, x') = b^{\geq} \hat{D}_m(x, x') + n^{\geq} \hat{D}_0(x, x') , \quad (24)$$

where the medium-induced part of the spectral function in contrast to  $\hat{D}_0$  is related to the classical absorption  $a$  as

$$2iq_0 \hat{D}_m(x, x') = ae^{iq_0(x-x')} . \quad (25)$$

Now Poynting's theorem (8) provides the incoherent energy flow  $S(L/2) = -S(-L/2)$  from the energy dissipation (21):  $2s(L/2) = -w$ . Then from (8) and (21), for any frequency  $\omega > 0$  and propagation direction  $\mathbf{q} = (q_0, \mathbf{q}_{\perp})$ , we obtain for the spectrally and directionally resolved energy flow

$$s = -\frac{i}{2q_0} (n^<\mathfrak{P}^> - n^>\mathfrak{P}^<) = (b - n)a . \quad (26)$$

In eq. (26),  $-na$  describes absorption ( $a > 0$ ) or emission ( $a < 0$ ) as the response of the medium to (and stimulated by) the given nonequilibrium distribution of external photons  $n$ . Putting  $n = 0$ , the emission  $ba$  is the response of the medium to vacuum fluctuations. It is to be regarded as spontaneous emission with respect to external photons, but it is stimulated by  $b$ . It is not directly related to the coherent absorption. Instead, the recombination  $i\mathfrak{P}^<$ , and so  $b$ , has to be calculated from the particle GF's of the interacting many-body system of field and matter on just the same footing as has been carried out here for the photon GF.

Assuming  $s = 0$ , i.e., vanishing energy flow between the medium and its surrounding, one arrives at  $b = n$ , which generalizes Kirchhoff's law to nonequilibrium, and  $b$  develops towards a Bose function  $b(\omega) = (\exp[\beta\hbar\omega] - 1)^{-1}$  for thermodynamic equilibrium.

If the externally given incoherent radiation field incident from outside is strong, i.e., for  $n \gg b$ , measuring the energy flow  $s$  would provide the same information as a classical (coherent) reflection-transmission experiment, namely the absorption  $a = s/n$ .

In the following, the opposite case  $b \gg n \rightarrow 0$ , i.e., emission  $s = ba$  into the surrounding vacuum is considered. Then the distribution  $b$  is accessible to direct observation in experiments measuring simultaneously the (incoherent) emission and the linear coherent absorption, i.e., transmission and reflection. It generalizes Planck's formula for the black body radiation to the nonequilibrium radiation of an excited medium in the steady state.

## QUASIEQUILIBRIUM

Of particular interest are those steadily excited states which can be regarded as quasi-equilibrium states. As such, e.g., for semiconductors, exciton gases generated at low up to moderate excitation and light-emitting diodes working at high excitation are to be mentioned. For quasi-equilibrium, due to the Kubo-Martin-Schwinger condition [8], the distribution  $b(\omega, \mathbf{q}_\perp)$  develops into a Bose distribution  $b(\omega) = (\exp[\beta(\hbar\omega - \mu)] - 1)^{-1}$ , being independent of  $\mathbf{q}_\perp$ . The chemical potential  $\mu$  starts at  $\mu = 0$  for complete thermal equilibrium and characterizes the degree of excitation beyond the thermal one for  $\mu > 0$ . Then the crossover from absorption ( $a > 0$ ) to gain ( $a < 0$ ) appears independently of  $\mathbf{q}_\perp$  at  $\hbar\omega = \mu$ , where the singularity in  $b$  is compensated by the zero in  $a$ . Hence, expanding  $b^{-1}$  and  $a$  at  $\hbar\omega = \mu$  yields that at crossover the emission stays finite and is given by the slope of the absorption according to  $s(\mu, \mathbf{q}_\perp) = k_B T \{\partial a(\omega, \mathbf{q}_\perp) / \partial \omega\}_{\hbar\omega=\mu}$ . Since both  $a = 1 - |r|^2 - |t|^2$  and  $b$  switch their signs, the emission  $s$  stays positive as it should be in the whole frequency region. Measuring  $s$  and  $a = 1 - |r|^2 - |t|^2$  would enable to check whether quasiequilibrium is realized. If so, the chemical potential  $\mu$  is fixed through the crossover point and, after that, the temperature and excitation density can be obtained directly from experimental data.

## LOW TEMPERATURE

For  $T \rightarrow 0$ , the Bose function degenerates to a step function and the emission  $s(\omega, \mathbf{q}_\perp) \rightarrow -\Theta(\mu - \hbar\omega)a(\omega, \mathbf{q}_\perp)$  vanishes completely in the absorption region  $\hbar\omega > \mu$  and reflects exactly the gain  $-a$  in the gain region  $\hbar\omega < \mu$ .

Concerning low temperatures, however, the theory given above needs to be supplemented if effects of quantum condensation occur. As such, the crossover from Bose-Einstein condensation of excitons at moderate excitation to the one of Cooper-like electron-hole pairs at high excitation has been addressed [9]. Its consequences for emission and absorption spectra will be discussed in detail in a forthcoming paper. However, some basic features should be commented on here. If quantum condensation occurs, an anomalous contribution to

$P_m^{\geq} \rightarrow P_m^{\geq} + P_{\text{cond}}\delta_{\mathbf{q}_\perp,0}\delta(x-x')\delta(\hbar\omega - \mu)$  will appear in addition to the normal generation  $iP_m^{\geq}$  and recombination  $iP_m^{\leq}$  considered above. The Kronecker  $\delta_{\mathbf{q}_\perp,0}$  indicates that this term is to be excluded from the  $\mathbf{q}_\perp$  integrals. The strength  $P_{\text{cond}}$  is determined by the fraction of quasiparticles in the condensate. Since it appears identically in both the generation  $iP_m^{\geq}$  and the recombination  $iP_m^{\leq}$ , its influence cancels in the classical absorption  $a$  according to (9) and (10). Consequently, those effects will not appear directly in classical absorption experiments, where at best they show up as smooth changes in the spectral shape of the absorption  $a$ . The same applies to the absorption ( $a > 0$ ) or emission ( $a < 0$ ) as the response of the medium to (and stimulated by) the externally given nonequilibrium distribution of photons  $n$  in (26).

However, an additional sharp peak in the emission

$$ba = \frac{1}{2q_0} i\mathfrak{P}^{\leq} \quad (27)$$

at  $\hbar\omega = \mu$ , whose strength is  $\propto P_{\text{cond}} \int dx \Theta(L/2 - |x|)|A(x)|^2$ , would give evidence for a condensate, since the normal part of the emission just at this frequency tends towards zero.

## CONCLUSION

Using a quantum-kinetic many-body approach, exact results have been presented for the interacting system of field and matter in a specified geometry. The spectral function of photons splits up into a vacuum-induced and a medium-induced contribution (see eqs. (16-18), for which the explicit expressions (20) and (25), respectively, have been obtained. It is noteworthy that both kinds of states are globally (i.e., inside *and* outside) defined and their split does not correspond to a spatial separation of the inside from the outside. There are, of course, many different options to split up the spectral function of photons, e.g., into the one of the pure vacuum  $\hat{D}_{\text{vac}}$  and the remaining. But only the decomposition according to eqs. (16-18) yields an exact cancellation of the medium-induced contribution in the balance (12) and so the physically clear and simple structure of the emission (26).

Also, according to eq. (24), the field fluctuations split up into a vacuum-induced and a medium-induced contribution whose strengths are given by the distributions  $n$  and  $b$ , respectively.

The distribution  $n$  describes the population of the vacuum-induced states and is externally given through preparation of the surrounding, e.g., either as a heat bath or by incoherent radiation incident from outside. Measuring the net energy flow  $-na$  induced by an incoherent radiation  $n$  incident from the outside provides the absorption  $a$  just as a classical (coherent) transmission-reflection experiment.

The distribution  $b$  describes the population of the medium-induced states. It is fixed by the steady-state excitation conditions of the medium and characterizes its globally defined transverse optical excitations as, e.g., excitonic polaritons in semiconductors. For a medium in thermal equilibrium,  $b$  tends towards a Bose function, to which a chemical potential can be attributed in the case of quasi-equilibrium. Thus, in a sense, real photons, i.e. after their interaction with the fermions of the medium is exactly considered, behave statistically like ideal bosons. E.g., for semiconductors, this applies likewise to optical excitations below (excitonic polaritons) and above the fundamental gap, whereas neither excitons nor ionized electron-hole pairs can be regarded as (ideal) bosons. For  $n = 0$ , i.e., without any preparation of the surrounding of the medium, emission is governed by the distribution  $b$ . It generalizes the Planck distribution for ideal photons in thermodynamic equilibrium to interacting ones in nonequilibrium.

Our results prove that even lasing can be regarded as quasi-thermal emission. This has already been demonstrated in ref. [2] neglecting spatial dispersion and is now exactly confirmed. All the novel features of this light result exclusively from an extremely strong renormalization of the globally defined coherent absorption  $a$  in the gain region due to high compensation of output losses there.

Emission out of a quantum condensate appears as an additional sharp peak at the crossover point, whereas no significant structures are expected in the coherent absorption.

As a challenge for both experimentalists and theorists, the nonequilibrium distribution  $b$  can be observed directly measuring emission and absorption simultaneously [11] or has to be computed, respectively, from the global recombination providing reasonable approximations for the polarization function.

It may appear counterintuitive and surprising that the medium-induced contribution  $\hat{D}_m$  exactly cancels out in the absorption and emission, and the question arises whether this remains true if one moves away from the restrictions to slab geometry and the steady state, or further, which form the energy flow law takes without them. The geometry-independence of Kirchhoff's and Planck's laws are bought with severe restrictions in other domains, namely to full thermal equilibrium and noninteracting photons. It can hardly be expected to obtain a radiation law which is universal in all these aspects and at the same time more detailed than just the condition of energy conservation, since quantities like absorption or distribution lose their meaning in a temporally inhomogeneous case. Moreover, in nonequilibrium, energy flow in both the medium and its environment have to be evaluated separately for the considered geometry, and the less

symmetries the geometry provides, the more difficult this will be. Taking this into account, it appears all the more surprising that for the present case of steady-state slab geometry the evaluation can be taken such a long way. What we know today is at least that the splitting of the spectral function in the presented form is a completely universal property of the photon GF [12] and will thus always affect Poynting's theorem. Analyzing the latter for a non-steady state, we suppose one will find the interplay of  $\hat{D}_m$  and  $\hat{D}_0$  differ and worth examining closer.

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